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PHYSICO-GEOMETRIC INVESTIGATION OF BRITTLE FRACTURE DURING CREEP*

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A geometrical model of brittle fracture during creep is proposed for metals that enables the qualitative dependence of the geometry of a typical microcavity in a polycrystal on the given stress level to be explained, and enables the distribution laws of the time up to specimen fracture to be found, that generalizes the Weibull law. The main idea underlying the model being proposed is the representation of the microcavity formation process in a material as a spontaneous disturbance of symmetry.

It is known that the brittle fracture mechanism during creep is the formation, growth, and coalescence of microcavities that result in macrocrack generation, which separates the specimen into parts. Under high-temperature creep conditions at fixed stresses commensurate with the material yield point at a given temperature, mainly micropores with sharp boundaries (so called w-pores) are observed while circular micropores (or r-pores) are observed in tests at lower stress levels /1/. In the latter case the length of the microcavity projection in any direction has identical order for different directions; consequently, the microcavities being formed are called micropores /2/. For w-pores the properties of a small body domain can be distinct in different directions depending on whether this direction is parallel to one of the microcavity faces.

The process of microcavity development under creep conditions results in a loss of local symmetry in the initial continuous medium. It is known /3/ that the transition from the symmetric to the asymmetric state at once is of low probability, while step-by-step passage is more probable when symmetry is partially conserved in the intermediate steps; the transition is here all the more probable at each stage, the less the symmetry is disturbed.

1. The initial symmetry group, i.e, the transformation group of the space G_0 under whose action local properties of the original undeformed medium are conserved, consists of translations, rotations, and similarities. The transition to the state that has a certain symmetry group $G \subset G_0$, i.e., the state all of whose properties are invariant under the action of transformations of a certain subgroup G of the group G_0 is most probable. Invariance of a state means, in particular, that the microcavity boundary does not change under the action of transformations from the group G: if a point a is on a microcavity boundary, this boundary also contains $g(a)$ for all transformations $g \in G$. Therefore, the boundary contains all points $g(a)$ for all $g \in G$, i.e., contains the orbit of the element a relative to the group G. The microcavity boundary thereby either coincides with the orbit of the group G, or consists of several such orbits. Consequently, to describe all possible kinds of microcavities it is sufficient to describe the orbits of subgroups of the group G_0.

The transition from the initial state from group G_0 to a state from group G is all the more probable the greater the symmetry in the group G, i.e., the greater the dimensionality of the group G (the dimensionality of a group is the minimal number of independent parameters needed to describe all the transformations from G). The greatest possible dimensionality of G is four /4/; a single orbit, a plane, corresponds to this dimensionality. The group G consists of similarity (one parameter), translations in the plane (two parameters), and rotations in the plane (one parameter). Two different symmetry groups corresponds to the value $\dim G = 3$ a group of rotations around a point (the orbit is a sphere), and the symmetry group of a line (translations along the line, rotations around it, and similarity). The second of these groups has just two orbits, the line and all the rest of space, consequently, the microcavity boundary cannot consist of such orbits. Corresponding to $\dim G = 2$ are the orbits: a half-plane, a circular cone, a right circular cylinder, etc.

Therefore, during creep the formation of microcavities with plane boundaries (boundaries corresponding to the greatest possible symmetry group) is most probable. At successive times of the creep process the formation of microcavities of approximately spherical shape is more probable. If fracture (the formation of a macrocrack seed) did not indeed occur at this stage, then the geometric model being proposed allows the appearance of microcavities of more complex shape.

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The time to fracture is not large for sufficiently high tensile stress levels and only microcavities with plane boundaries (w-pores) succeed in being formed. For lower stress levels the specimen "lifetime" increases by several orders, consequently, the microcavities are smoothed out because of random processes in the material microstructure, and circular micropores (r-pores) are formed. Microcavities of more complex shape have not been observed experimentally in metals /2/.

The coalescence of microcavities can also be considered from the viewpoint of a spontaneous disturbance of symmetry. The union of two geometric shapes corresponding to microcavities has a smaller symmetry group, in general, than each of them. Hence, according to the general procedure of a spontaneous disturbance of symmetry, it follows that coalescence of microcavities yielding a smaller disturbance of symmetry is most probable.

Thus, the union of a sphere with a plane has a one-dimensional symmetry group, the union of a plane with a plane has a two-dimensional group, and the union of two spheres has a one-dimensional symmetry group. Therefore, unions of a plane with a plane, and a sphere with a sphere are more probable, resulting in a more probable coalescence of microcavities of w- or r-kinds with the same kind of microcavities. Therefore, circular pores are more often interconnected, and not with cavities with sharp boundaries in a real metal specimen where pores of both kinds exist at a certain time of the creep process.

2. Let us use the geometrical results obtained to give a quantitative description of brittle fracture during creep.

The processes of microcavity formation, growth, and coalescence in a material are the crux of the latent stage of the fracture process during creep /5/. As a rule, a scalar damage parameter ω is used to describe fracture at this stage. At the macrolevel ω characterizes "loosening" of the material resulting from the microcavity formation, growth and coalescence processes. A fundamental estimation of material loosening at the macrolevel is the relative change in material density, which does not ordinarily exceed 1-2% for the whole specimen /6/.

According to usual representations of the mechanics of a continuous medium /7/, if a specimen is partitioned into domains (subvolumes) Δ_k $(k = 1, 2, \ldots, N)$ whose size is much less than the specimen size and much greater than the material microinhomogeneity at a certain time of the deformation process, the fracture processes in each of the subvolumes are independent of analogous processes in the other subvolumes. In conformity with the brittle fracture mechanism noted during metal creep, the probability $p(\Delta_k)$ of fracture of each subvolume Δ_k under given external conditions is determined uniquely by the value of the damage parameter $\omega(\Delta_k)$ for this subvolume $p(\Delta_k) = F(\omega(\Delta_k))$, where F is a certain function whose form depends on the external conditions (stress and temperature).

Following /7/, we use the principle of the "weakest link" to describe the relation between specimen fracture and fracture of the individual subvolumes: fracture of a certain domain occurs if and only if its most defective part is fractured. In the case of fracture due to microcavity accumulation, this principle means that the subvolumes are fractured due to the most loosened of their parts. Consequently, a specimen is not fractured in just that case when none of its subvolumes is fractured. And since the processes are independent in the subvolumes, the probability that a specimen will not be fractured until a time t equals the product of the probabilities for the subvolumes. If $P(t)$ is the probability that macrofracture of a body under a given external load will be start at a time less than t, we obtain that

$$1 - P(t) = \prod_{k=1}^{N} (1 - p(\Delta_k)) = \prod_{k=1}^{N} (1 - F(\omega(\Delta_k))) \tag{2.1}$$

We will use the physical nature of this parameter to determine $\omega(\Delta_k)$. Material loosening appears at the macrolevel in the form of a residual change in the material volume, whose fundamental role for the cold (athermal) plastic deformation processes was established by Novozhilov /8/. Since the residual change in the volume is much more significant in creep processes, it is natural to take an increasing function of the relative inelastic change in material volume ε_v^p as the damage parameter. It is most convenient /9/ to use the expression $\omega(\Delta_k) = \ln(1 + \varepsilon_v^p)$, for ω, where ε_v^p is the inelastic part of ε_v, $\varepsilon_v = (\Delta v_k - \Delta v_{k0})/\Delta v_{k0}$, Δv_k is the volume of the element Δ_k at a certain time of the process and Δv_{k0} is the volume of the same element at the initial time. We have $\omega \approx \varepsilon_v^p$ for $(\varepsilon_v^p)^2 \ll 1$.

Thus, the fracture process for fixed external conditions is determined uniquely by the change in the material volume. In turn, this quantity is determined by the microcavity statistics, i.e., their size distribution and is indepdent of the spatial microcavity distribution. Consequently, the simplifying assumption that such a distribution is identical for all subvolumes of the specimen under consideration can be used in computing the microcavity size distribution. With this assumption, for sufficiently large Δv_k and a quantity ε_v^p determined by the microcavity statistics in the domain Δ_k, approximately identical for

all the subdomains we have $\Delta v_k \approx \Delta v_{k0}(1 + \varepsilon_v^p)$ Hence, summing over k we have $v \approx v_0(1 + \varepsilon_v^p)$, where v is the specimen volume at the current time of the process, and v_0 is its initial volume. The value of ε_v^p for the whole body is approximately identical with the value of ε_v^p for the subvolumes; consequetly, even the damage parameter for the whole body is approximately equal to $\omega(\Delta_k)$. Then (2.1) results in the value $P(t) = 1 - (1 - F(\omega))^{N_s}$ where ω is the damage parameter for the whole specimen. If $\langle \Delta v \rangle$ is the mean volume of the subvolumes, then $N = v / \langle \Delta v \rangle$ from which it follows that

$$P(t) = 1 - \exp(-vf(\omega)), \quad f(\omega) = -\ln(1 - F)/\langle \Delta v \rangle \tag{2.2}$$

The value of the damage depends, in turn, on time. Substituting this dependence into (2.2), we obtain that for a certain function $\varphi(t) = f(\omega(t))$

$$P(t) = 1 - \exp(-v\varphi(t)) \tag{2.3}$$

Consequently, it is sufficient to find $\varphi(t)$ to determine the fracture time statistics, i.e., $P(t)$.

3. We will consider first the case when mainly ω-pores and accumulated in the material during creep. The formation of ω-pores corresponds to that spontaneous disturbance of symmetry for which symmetry relative to the change in the length scale is not disturbed. As remarked in Sect.1, invariance relative to similarity is conserved locally during ω-pore formation. Consequently, the values of all the dimensionless combinations of the body characteristics should be independent of what units these characteristics are measured in. We will apply this principle thrice.

For each volume v the time in which the probability of fracture takes a certain given value P_0, i.e., $\exp(-v\varphi(t)) = 1 - P_0$, is denoted by $t(v)$, or

$$t(v) = \varphi^{-1}(-\ln(1 - P_0)/v) \tag{3.1}$$

For any real number l the characteristic $t(lv)/t(v)$ is dimensionless. The passage to an m times smaller length unit transfers v into m^3v and the scale invariance remarked results in the equality $t(lv)/t(v) = t(lm^3v)/t(m^3v)$ for any $l > 0, m > 0$, meaning that the ratio $t(lv)/t(v)$ depends only on l and is independent of v, i.e.

$$t(lv) = t(v)\, g(l) \tag{3.2}$$

for a certain function $g(l)$. To solve the functional Eq.(3.2), we substitute therein first $x = l, y = v$ and then $x = v, y = l$ and we obtain that $t(xy) = t(x)\, g(y) = t(y)\, g(x)$ for any $x > 0, y > 0$, from which $g(x)/t(x) = g(y)/t(y) = \text{const}$, or $g(x) = \text{const}\, t(x)$ and (3.2) takes the form $t(lv) = \text{const}\, t(l)\, t(v)$. Multiplying both sides of this relationship by const and using the notation $t_0(z) = \text{const}\, t(z)$, we obtain $t_0(lv) = t_0(l)\, t_0(v)$. As is known /10/, the solution of this equation has the form $t_0 v \sim v^s$, where s is a constant; consequently, $t_0(v) = \text{const}\, v^s$. Substituting this expression into (3.1) we have $\varphi(t) = C_0 t^n$, $n = -1/s$, $C_0 = (\text{const})^n \ln(1 - P_0)$ and for such a function $\varphi(t)$ from (2.3) there follows the Weibull distribution law of the time to specimen fracture

$$P(t) = 1 - \exp(-A_1 t^n), \quad A_1 = C_0 v \tag{3.3}$$

Considering the quantity $t(l\Delta v)/t(\Delta v)$, analogously, where $\Delta v = \omega v_0$ is the total volume of all the microcavities in the domain Δ, we obtain that Δv, meaning also the damage parameter ω, depends on the time as a power law: $\omega = \Omega t^\alpha$ /2/. It hence follows that $f(\omega)$ also depends on ω as a power law. In fact, the equality $\varphi(t) = f(\omega(t))$ holds for all t, where $\varphi(t) = C_0 t^n$. To find the value of $f(\omega)$ for aribitrary $\omega > 0$ we find a t such that $\omega(t) = \Omega t^\alpha$ (this t is determined by the formula $t = (\omega/\Omega)^{1/\alpha}$). Then

$$f(\omega) = \varphi(t(\omega)) = C_1 \omega^{s_1}, \quad s_1 = n/\alpha, \quad C_1 = C_0 \Omega^{-s_1} \tag{3.4}$$

Considering the dimensionless quantity $p(lr)/p(r)$, where $p(r)\, dr$ is the fraction of micropores of radius from r to $r + dr$ on the speciment cut, we obtain that the microcavity diameter distribution should also be described by the power law $p(r) = C_2 r^{-s_2}$.

4. We will now investigate the case when brittle fracture in a metal material occurs due to the formation of r-pores. As is shown in Sect.1, in this case the spontaneous disturbance of symmetry resulting in r-pore formation also disturbs the invariance with respect to similarity. Consequently, in such a situation the scale invariance model described in Sect.3 can be considered as a first approximation to a description of the fracture process. The inadequacy of the first approximation is manifest, say, in the fact that the experimentally determined microcavity diameter distribution law differs substantially from the power law described in Sect.3 for their small values. The scale-invariant model of the first approximation in which the dependence of ω on t and of p on r is a power law, is equivalent to a linear dependence of $\ln\omega$ on $\ln t$ and of $\ln p$ on $\ln r$, i.e.,

$$\omega = \Omega t^{\alpha + \beta \ln t} \tag{4.1}$$

$$p(r) = \exp[-(A \ln^2 r + B \ln r + C)] \tag{4.2}$$

where $\alpha, \beta, \Omega, A, B, C$ depend on the external test conditions. Since the material damageability during creep is cumulative, the relation $\omega'(t) > 0$ should be satisfied up to the time of fracture; consequently $\beta \leqslant 0$.

Strictly speaking, (4.1) is inapplicable for large t since $\omega(t)$ starts to decrease here. This means that for such t the quadratic terms are inadequate, and terms containing expressions cubic in $\ln t$ must still be taken into account. However, analysis of the experimental data shows that the approximation (4.1) is adequate for describing the real fracture process.

The damageability ω is defined essentially as the ratio of the volume occupied by microcavities to the specimen volume. Consequently, if a random point of the specimen is taken (selected in conformity with the uniform probability distribution law for which the probability falls into some domain proportional to its volume), the probability that this point will fall within a microcavity equals the damageability ω. This same probability can be estimated differently by using a plane microcavity size distribution rather than the volume distribution. In fact, the random selection described above can be realized in two stages: first the oriented plane is chosen randomly, and then the random point in the appropriate plane section. A plane cut essentially indicates selection of a randomly oriented plane (randomly oriented relative to the microcavities). Consequently, the damageability ω equals the probability that a randomly selected point on a randomly oriented cut will drop into a microcavity. Indeed, test results do not contain microcavity size distribution statistics for different plane cuts /2, 6/.

Let us show how the statistics corresponding to just one section can be utilized. All the considerations are carried out within the framework of a model in which the microcavity volume distribution is assumed to be homogeneous and isotropic. Consequently, the microcavity size distribution law on a plane cut, in particular, is independent of the orientation of the plane over which the cut is made. In particular, this means ω equals the probability that a point randomly selected in the plane drops into a microcavity on that very cut on which there is statistics (from test). The probability is, in turn, equal to the ratio between the total area of the microcavities incident in the domain of the cut, and its area S. If $p_S(r)$ is the microcavity size distribution density on the plane cut, this ratio takes the form

$$\omega = \pi S^{-1} \int_0^\infty p_S(r)\, r^2\, dr$$

Arguments yielding the foundation of (4.2) result in an analogous formula for the dependence $p_S(r)$ also. We substitute it into the last integral, after evaluating which we obtain

$$\omega = \pi^{1/2} (AS)^{-1} \exp\left(C - (B-3)^2/(4A)\right) \tag{4.3}$$

Experimental data on the micropore radius distribution are actually in agreement with a log-normal law /11/.

To determine $P(t)$ we use relationship (2.2) in which $\omega(t)$ is given by the expression (4.1).

The investigation is performed within the framework of the physical model, according to which the fracture probability is determined by only one scalar parameter ω under given external conditions. Consequently, the function $f(\omega)$ describing the dependence of $P(t)$ on ω in (2.2) is also defined solely by the external conditions and is independent of the microcavity form. It is shown in Sect.3 in the example of fracture due to w-pores that $f(\omega)$ is a power-law function. Therefore, even in the case of r-pores expression (3.4) can be utilized. Substituting it and $\omega(t)$ into (2.2), we obtain the following Weibull law modification that is valid for $t < \alpha/(2\,|\,\beta\,|)$:

$$P(t) = 1 - \exp\left(-A_1 t^{n+n_1 \ln t}\right), \quad n = s_1\alpha, \quad n_1 = s_1\beta \tag{4.4}$$

As already noted, (4.4) is not applicable for $t > \alpha/(2\,|\,\beta\,|)$ (terms of the next orders in $\ln t$ must be taken into account for such t), but in practice (see the Appendix), the probability that fracture will not occur up to this critical time is small and can be negleted completely.

In the model under consideration n_1 describes the difference of the distribution law from the Weibull law corresponding to scale invariance. In coformity with the singularities of the brittle frature process during creep the scale invariance is disturbed for small stresses σ relative to the yield point at a given temperature. (Consequently $n_1 \approx 0$ for large σ, while tendency for the value of n_1 to grow holds as σ decreases.

5. We will now examine the dependence of the parameters in (3.3), (4.1), (4.4) on a given stress.

If the fracture process is governed by ω-pores (scale invariant case), then $\ln \omega = \ln \Omega +$

$d \ln t$, where $\ln \Omega$ and α depend on σ for a fixed testing temperature. In this case the combination $f(\omega(l\sigma, t))/f(\omega(\sigma, t))$ must be added to the invariant dimensionless combinations utilized in Sect.3. Its analysis results in the power-law dependence of $f(\omega(\sigma, t))$ on σ, and $f(\omega)$ according to (3.4) is also a power-law function. Therefore, $\omega(\sigma, t)$ depends in a power-law manner on σ for a fixed t, i.e., the function $\ln \omega$ is linear in each of the variables $\ln \sigma$, $\ln t$, and for all $\sigma > 0$, $t > 0$ the formula

$$\ln(\omega) = a + b \ln \sigma + (c + d \ln \sigma) \ln t \tag{5.1}$$

holds, where $a + b \ln \sigma = \ln \Omega$, $c + d \ln \sigma = \alpha$.

Substituting (5.1) into (2.2), we obtain that the Weibull law parameters A_1 and n depend on σ as follows: $n = s_1(c + d \ln \sigma)$, $A_1 = vC_1 \exp(s_1 a + s_1 b \ln \sigma)$.

If mainly r-pores are observed during fracture, then considering (5.1) as a first approximation as in Sect.4, we obtain that

$$\ln \omega = a + b \ln \sigma + (c + d \ln \sigma) \ln t + (c_1 + d_1 \ln \sigma) \ln^2 t$$

plus terms of higher orders in $\ln \sigma$ and $\ln t$. Comparing this expression with $\ln \omega$ from the logarithmic relationship (4.1), we have the dependence of the parameter on the stress level: $\beta = c_1 + d_1 \ln \sigma$. The dependence of n_1 on σ: $n_1 = s_1(c_1 + d_1 \ln \sigma)$ follows from (4.4). As $\sigma \to 0$ we have $|n_1| \to +\infty$ from the last relationship, i.e., the difference between (4.4) and the Weibull law is a maximum for small σ. This difference diminishes as the given stress level increases.

6. A method of predicting the probability of specimen fracture at different times follows from the model proposed above. In conformity with (4.1) it is sufficient to find three parameters in the dependence of the damage ω on the time t for this. It is necessary to estimate $\omega(t_i)$ at not less than three instants of time t_i and to find α, β, and $\ln \Omega$ as coefficients in the quadratic dependence $\ln \omega(t_i)$: $\ln \omega(t_i) = \ln \Omega + \alpha \ln t_i + \beta \ln^2 t_i$. In turn, it is necessary to utilize (4.3) to compute $\omega(t_i)$. Consequently, to evaluate $\omega(t_i)$ it is necessary to obtain $p(r)$ the microcavity size distribution, at the specimen cut, and then to estimate A, B, C as quadratic parameters in $\ln r$ in the dependence for $\ln p(r)$.

It should be noted that the physico-geometrical investigation performed above refers to the case of body fracture with homogeneous fields ε_v^p and ω. In the general case, the microcavity distribution can be considered to be locally-homogeneous, i.e., power-law or log-normal, but the parameters of the appropriate distribution laws are already, generally speaking, dependent on points of the body, i.e., from physical fields.

7. *Appendix.* Results of measuring the micropore concentration as a function of the radius were known in a section of specimens of 304 stainless steel with a 40-50 μm grain size made at three different times of the creep process and at a time preceding specimen fracture (the test conditions were $\sigma = 63$ MPa, and $T°C = 700°$ /11/).

Analysis of these experimental data showed that the three distribution functions $p(r)$ are described well by the log-normal law (4.2). Values of A, B, C found to 0.1 accuracy are presented below, where $t = 5 \times 10^5$ sec corresponds to the time of fracture, and results are also presented of calculations of the damage parameter for the whole specimen, found as the total area of all micropores in the section S from (4.3). For $t = 0.6 \times 10^5$ sec, $A = -1.9$, $B = -1.3$, $C = -9.1$, $\omega S = 6.6 \times 10^3$ μm², for $t = 1.3 \times 10^5$ sec, $A = 4.5$, $B = 0.6$, $C = 3.3$, $\omega S = 42 \times 10^3$ μm², for $t = 2.6 \times 10^5$ sec, $A = 2.8$, $B = 0.3$, $C = 3.3$, $\omega S = 130 \times 10^3$ μm², and for $t = 5.0 \times 10^5$ sec, $A = 3.3$, $B = 0.6$, $C = 3.0$, and $\omega S = 180 \times 10^5$ μm².

According to the data in /11/, $\ln \omega$ depends quadratically on $\ln t$, i.e., $\omega(t)$ is described by a log-normal law. This example thereby confirms the validity of (4.1), (4.2), and (4.4) and illustrates the development of the model of brittle fracture under creep.

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COMPLETE CONTROLLABILITY OF LINEAR DYNAMIC SYSTEMS*

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A complete controllability criterion is suggested for a linear dynamic system with bounded controls. It is shown that programmed control, taking the system from one state to another, can be constructed in quasipolynomial form. The problem of constructing such a control thus basically reduces to solving a linear system of equations.

1. One of the fundamental results of control theory is the Kalman criterion /1/, which provides the necessary and sufficient conditions of complete controllability of dynamic systems of the form

$$x^{\cdot} = Ax + Bu,\ x \in \mathbf{R}^n,\ u \in \mathbf{R}^m \tag{1.1}$$

Here, $A: \mathbf{R}^n \to \mathbf{R}^n$, $B: \mathbf{R}^m \to \mathbf{R}^n$ are time-independent linear operators. The Kalman criterion states that the pair of matrices A, B should satisfy the following condition of general position:

$$\operatorname{rank}(B,\ AB,\ \ldots,\ A^{n-1}B) = n \tag{1.2}$$

This condition ensures that any point $x_0 \in \mathbf{R}^n$ is reachable from any point $x_1 \in \mathbf{R}^n$ by moving along a trajectory of the dynamic system (1.1) with some control $u = u(t)$.

In this paper, we present an analogue of the Kalman criterion for the case when the controls u in Eq.(1.1) are bounded,

$$|u| \leqslant C \tag{1.3}$$

and we also provide a technique for constructing a programmed control that achieves a transition between states. Given the constraint (1.3), the Kalman condition (1.2) is of course totally insufficient for complete controllability. Indeed, if all the eigenvalues of the matrix A have strictly negative real parts, then starting from any point $x_0 \in \mathbf{R}^n$ and moving along the trajectories of the system (1.1), (1.3), we will never be able to leave a certain bounded set, regardless of the choice of the matrix B. If conversely all the eigenvalues of the matrix A have strictly positive real parts, then, say, $0 \in \mathbf{R}^n$ is unreachable from a sufficiently distant point $x_0 \in \mathbf{R}^n$.

2. Let us discuss the following theorem, which was first proved in /2/ (for some further results, see /3, 4/).

Theorem 1. For complete controllability of system (1.1), (1.3), it is necessary and sufficient that, in addition to the Kalman condition (1.2), we also have

$$\operatorname{Re}\lambda_i = 0 \tag{2.1}$$

where λ_i are the eigenvalues of the matrix A.

Let us explain the need for condition (2.1). To fix our ideas, assume that the matrix A has the eigenvalue λ and

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